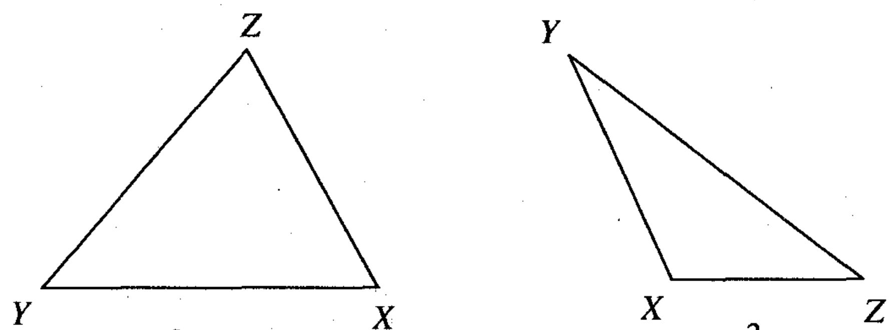
So we have three results: the Pythagorean theorem for a right angle, and the two new results for an acute and an obtuse angle. Just as with the sine function, we can make all these results into a single formula, the so-called *Law of Cosines*: In triangle ABC,

$$b^2 = a^2 + c^2 - 2ac\cos B\,.$$

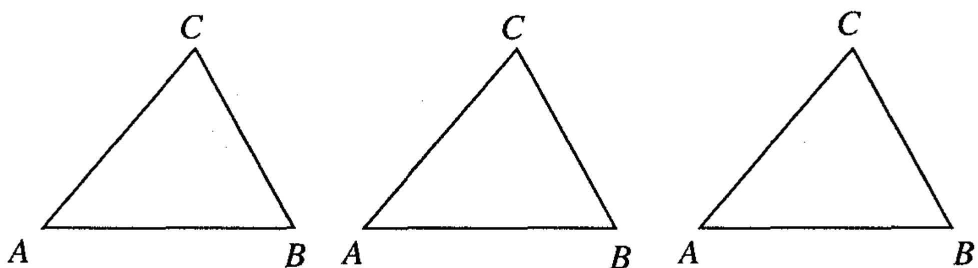
Exercises

1. Check to see that this is correct, whether angle B is acute, right, or obtuse.

2. In each of the triangles below, use the Law of Cosines to express the square of the indicated side in terms of the other two sides and their included angle:



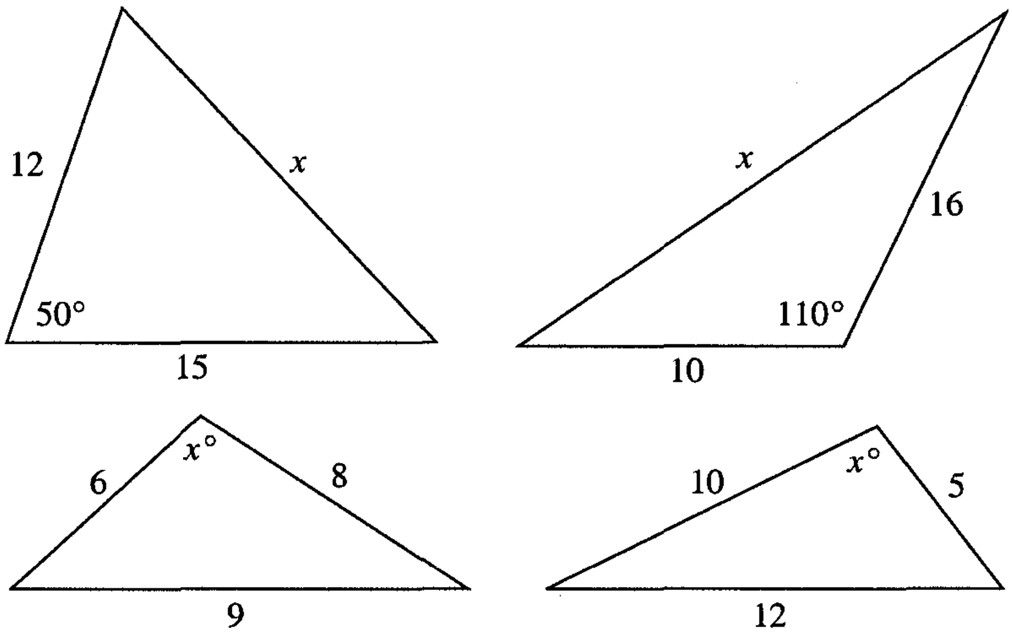
$x^2 =$ $\qquad$ $x^2 =$



$a^2 =$ $\qquad$ $b^2 =$ $\qquad$ $c^2 =$

3. We know, from geometry, that a triangle is determined by SAS (the lengths of two sides and the angle between them). Explain how the Law of Cosines allows us to calculate the missing parts of a triangle, if we are given SAS.

4. Find the side or angle marked x in each diagram below:



5. In triangle ABC, $AB = 10$, $AC = 7$, and $BC = 6$. Find the measures of each angle of the triangle.

6. Peter's teacher gave the following problem:

   > A parallelogram has sides 3 and 12. Find the sum of the squares of its diagonals.

   But Peter had trouble even drawing the diagram. He knew that opposite sides of a parallelogram are equal, so he knew where to put the numbers 3 and 12. But then he didn't know what kind of parallelogram to draw. He drew a rectangle (which, he knew, is a kind of parallelogram). Then he drew a parallelogram with a 30° angle, and another parallelogram with a 60° angle. But he didn't know which one to use to do the computation.

   Can you help Peter out?

7. Show that the sum of the squares of the sides of any parallelogram is equal to the sum of the squares of the diagonals.

8. If M is the midpoint of side BC in triangle ABC, then AM is called a median of triangle ABC. Show that for median AM, $4AM^2 = 2AB^2 + 2AC^2 - BC^2$.

   Hint: The diagram for this problem is "half" of the diagram for Exercise 7 above.

9. Show that the sum of the squares of the three medians of a triangle is 3/4 the sum of the squares of its sides.

10. The diagonals of quadrilateral $ABCD$ intersect inside the figure. Show that the sum of the squares of the sides of the quadrilateral is equal to the sum of the squares of its diagonals, plus four times the length of the line segment connecting the midpoints of the diagonals (notice that this generalizes problem 6).

11. In triangle ABC, angle C measures 60 degrees, $a = 1$ and $b = 4$. Find the length of side c.

12. In triangle ABC, angle C measures 60 degrees. Show that $c^2 = a^2 + b^2 - ab$. What is the corresponding result for triangles in which angle C measures 120 degrees?

13. Three riders on horseback start from a point X and travel along three different roads. The roads form three 120° angles at point X. The first rider travels at a speed of 60 MPH, the second at a speed of 40 MPH, and the third at a speed of 20 MPH. How far apart is each pair of riders after 1 hour? After 2 hours?

Appendix – Three big ideas and how we can use them

I. Invariants: Motions in the plane

We often talk about the congruence of triangles. Two triangles are congruent if one can be moved so that it fits exactly on the other. So we can say that two congruent triangles are exactly the same, except for their position.

The two triangles below cannot be considered congruent if we confine our motions to the plane. To move one of them onto the other, we must flip it around (reflect it in a line) before we can make it fit. These triangles are *mirror images* of each other.